The energy of the universe in teleparallel gravity

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Abstract. The teleparallel versions of the Einstein and the Landau-Lifshitz energy-momentum complexes of the gravitational field are obtained. By using these complexes, the total energy of the universe, which includes the energy of both the matter and the gravitational fields, is then obtained. It is shown that the total energy vanishes independently of both the curvature parameter and the three dimensionless coupling constants of teleparallel gravity.

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1. Introduction

The notion of energy and/or momentum localization of the gravitational field is one of the oldest and most controversial problems of the general theory of relativity [1]. Following the energy-momentum pseudotensor of Einstein [2], several other prescriptions have been introduced, leading to a great variety of expressions for the energy-momentum pseudotensor of the gravitational field [3]. These pseudotensors are not covariant objects because they inherently depend on the reference frame, and thus cannot provide a truly physical local gravitational energy-momentum density. The physical origin of this difficulty lies in the principle of equivalence, according to which a gravitational field should not be detectable at a point. Consequently, the pseudotensor approach has been largely questioned, although never abandoned. A quasilocal approach has been proposed and has been widely accepted [4]. However, it has been shown by Bergqvist [5], that there are an *infinite number* of quasilocal expressions for the gravitational field. Still in this context, it has been shown recently by Chang and Nester [6] that every energy energy-momentum pseudotensor can be associated with a particular Hamiltonian boundary term. In this sense it is usually said that the quasilocal energy-momentum rehabilitates the pseudotensor approach. A natural question then arises: What role does this gravitational energy density play in the description of the total energy of the universe? For example, during inflation the vacuum energy driving the accelerated expansion of the universe, and which was responsible for the creation of radiation and matter in the universe, is drawn from the energy of the gravitational field [7]. Such transition of gravitational energy into of a "cosmological fluid" must face problems of localization on account of the problems discussed above. Despite this difficulty, there has been several attempts [8, 9, 10] to calculated the total energy of the expanding universe. In one of these attempts the Einstein energy-momentum pseudotensor has been used to represent the gravitational energy [9], which led to the result that the total energy of a closed Friedman-Robertson-Walker (FRW) universe is zero. In another attempt, the symmetric pseudotensor of Landau-Lifshitz has been used [10]. In [11], the total energy of the anisotropic Bianchi models has been calculated using different pseudotensors, leading to a similar result. Several other attempts, using Killing vectors, or using the conservation law in the vierbein formulation, also led to the same conclusion. Recently [12], it has been shown that open, or critically open FRW universes, as well as Bianchi models evolving into de Sitter spacetimes also have zero total energy.

An alternative approach to gravitation is the so called teleparallel gravity [13], which corresponds to a gauge theory for the translation group based on the Weitzenböck geometry [14]. In this theory, gravitation is attributed to torsion [15], which plays the role of a force [16], and the curvature tensor vanishes identically. The fundamental field is represented by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial part of the tetrad field, and thus induces on spacetime a teleparallel structure which is directly related to the presence of the gravitational field. The interesting point of teleparallel

gravity is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the case, for example, of the energy-momentum problem, which becomes more transparent when considered from the teleparallel point of view. In fact, it has been shown recently that the energy-momentum gauge current associated to the teleparallel gravity is a true tensor that reduces to the Møller's canonical energy-momentum density of the gravitational field when returning to the geometrical approach [17].

By working then in the context of teleparallel gravity, the basic purpose of this paper will be to obtain the teleparallel version of both the Einstein and Landau-Lifshitz energy-momentum complexes. As an application, the energy of a FRW universe will be calculated, which includes the energy of matter, as well as the energy of the gravitational field. It will be shown that the total energy vanishes independently of the curvature parameter and the three dimensionless coupling constants of teleparallel gravity. We will proceed according to the following scheme. In section 2, we review the main features of teleparallel gravity, and obtain the teleparallel versions of Einstein and Landau-Lifshitz complexes. In section 3, it is obtained the tetrad field, the non-zero components of the Weitzenböck connection, the torsion tensor, and the superpotentials for the FRW universe in Cartesian coordinates. Discussions and conclusions are presented in section 4.

2. Teleparallel gravity

In teleparallel gravity, spacetime is represented by the Weitzenböck manifold W^4 of distant parallelism. This gravitational theory naturally arises within the gauge approach based on the group of the spacetime translations. Accordingly, at each point of this manifold, a gauge transformation is defined as a local translation of the tangent-space coordinates,

$$x^a \rightarrow x'^a = x^a + b^a$$

where $b^a = b^a(x^\mu)$ are the transformation parameters. For an infinitesimal transformation, we have

$$\delta x^a = \delta b^c P_c \, x^a,$$

with δb^a the infinitesimal parameters, and $P_a = \partial_a$ the generators of translations. Denoting the translational gauge potential by $A^a{}_\mu$, the gauge covariant derivative for a scalar field $\Phi(x^\mu)$ reads [16]

$$D_\mu \Phi = h^a{}_\mu \partial_a \Phi, \tag{1}$$

where

$$h^a{}_\mu = \partial_\mu x^a + A^a{}_\mu \tag{2}$$

is the tetrad field, which satisfies the orthogonality condition

$$h^a{}_\mu \, h_a{}^\nu = \delta_\mu^\nu. \tag{3}$$

This nontrivial tetrad field induces a teleparallel structure on spacetime which is directly related to the presence of the gravitational field, and the Riemannian metric arises as

$$g_{\mu\nu} = \eta_{ab}\, h^a{}_\mu\, h^b{}_\nu\ . \tag{4}$$

In this theory, the fundamental field is a nontrivial tetrad, which gives rise to the metric as a by-product. The parallel transport of the tetrad $h^a{}_\mu$ between two neighbouring points is encoded in the covariant derivative

$$\nabla_\nu h^a{}_\mu = \partial_\nu h^a{}_\mu - \Gamma^\alpha{}_{\mu\nu} h^a{}_\alpha, \tag{5}$$

where $\Gamma^\alpha{}_{\mu\nu}$ is the Weitzenböck connection. Imposing the condition that the tetrad be parallel transported in the Weitzenböck space-time, we obtain

$$\nabla_\nu h^a{}_\mu = \partial_\nu h^a{}_\mu - \Gamma^\alpha{}_{\mu\nu} h^a{}_\alpha \equiv 0.$$

This is the condition of absolute parallelism, or teleparallelism [15]. It is equivalent to

$$\Gamma^\alpha{}_{\mu\nu} = h_a{}^\alpha \partial_\nu h^a{}_\mu \tag{6}$$

which gives the explicit form of the Weitzenböck connection in terms of the tetrad, and

$$T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} \tag{7}$$

is the torsion of the Weitzenböck connection. As we already remarked, the curvature of the Weitzenböck connection vanishes identically as a consequence of absolute parallelism.

The action of teleparallel gravity in the presence of matter is given by

$$S = \frac{1}{16\pi G}\int d^4x\, h\, S^{\lambda\tau\nu}\, T_{\lambda\tau\nu} + \int d^4x\, h\, \mathcal{L}_M \tag{8}$$

where $h = \det(h^a{}_\mu)$, $\mathcal{L}_M$ is the Lagrangian of the matter field, and $S^{\lambda\tau\nu}$ is the tensor

$$S^{\lambda\tau\nu} = c_1 T^{\lambda\tau\nu} + \frac{c_2}{2}\left(T^{\tau\lambda\nu} - T^{\nu\lambda\tau}\right) + \frac{c_3}{2}\left(g^{\lambda\nu}\, T^{\sigma\tau}{}_\sigma - g^{\tau\lambda}\, T^{\sigma\nu}{}_\sigma\right). \tag{9}$$

with c_1, c_2, and c_3 the three dimensionless coupling constants of teleparallel gravity [15]. For the specific choice

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1, \tag{10}$$

teleparallel gravity yields the so called teleparallel equivalent of general relativity.

By performing variation in (8) with respect to $h^a{}_\mu$, we get the teleparallel field equations,

$$\partial_\sigma(hS_\lambda{}^{\tau\sigma}) - 4\pi G\,(ht^\tau{}_\lambda) = 4\pi G\, h\, T^\tau{}_\lambda, \tag{11}$$

where

$$t^\tau{}_\lambda = \frac{1}{4\pi G} h\Gamma^\nu{}_{\sigma\lambda} S_\nu{}^{\tau\sigma} - \delta^\tau{}_\lambda \mathcal{L}_G \tag{12}$$

is the energy-momentum pseudotensor of the gravitational field [17]. Rewriting the teleparallel field equations in the form

$$h(t^\tau{}_\lambda + T^\tau{}_\lambda) = \frac{1}{4\pi G}\partial_\sigma(hS_\lambda{}^{\tau\sigma}), \tag{13}$$

as a consequence of the antisymmetry of $S_{\lambda}{}^{\tau\sigma}$ in the last two indices, we obtain immediately the conservation law

$$\partial_{\tau}[h(t^{\tau}{}_{\lambda} + T^{\tau}{}_{\lambda})] = 0. \tag{14}$$

For the particular choice (10) of the parameters, on account of the identity

$$\partial_{\sigma}(hS_{\lambda}{}^{\tau\sigma}) - 4\pi G\ (ht^{\tau}{}_{\lambda}) \equiv \frac{h}{2}\ \left(R^{\tau}_{\lambda} - \frac{1}{2}\delta^{\tau}{}_{\lambda}R\right), \tag{15}$$

the teleparallel field equation is the same as Einstein's equation [16]. Using this equivalence, as well as Eq. (13), we find that $hS_{\lambda}{}^{\tau\sigma} = U_{\lambda}{}^{\tau\sigma}$ coincides with Freud's superpotential. Consequently, $t^{\tau}{}_{\lambda}$ is nothing but the teleparallel version of Einstein's gravitational energy-momentum pseudotensor. This superpotential and the Lagrangian $\mathcal{L}_G$ of the gravitational field are related by

$$U_{\lambda}{}^{\tau\sigma} = 4\pi G h^{a}{}_{\lambda}\frac{\partial\mathcal{L}_G}{\partial(\partial_{\sigma}h^{a}{}_{\tau})}\,. \tag{16}$$

Equation (13), therefore, can be rewritten as

$$h\ \mathcal{T}_{E}{}^{\tau}{}_{\lambda} = \frac{1}{4\pi G}\partial_{\sigma}(U_{\lambda}{}^{\tau\sigma}), \tag{17}$$

where $\mathcal{T}_{E}{}^{\tau}{}_{\lambda}$ is the Einstein energy-momentum complex, which is given by the divergence of the Freud's superpotential. The Bergmann-Thompson energy-momentum complex, on the other hand, is

$$h\ \mathcal{T}_{BT}{}^{\mu\tau} = \frac{1}{4\pi G}\partial_{\sigma}(g^{\mu\lambda}U_{\lambda}{}^{\tau\sigma}), \tag{18}$$

whereas the Landau-Lifshitz complex is

$$h\ \mathcal{T}_{LL}{}^{\mu\tau} = \frac{1}{4\pi G}\partial_{\sigma}(h\ g^{\mu\lambda}U_{\lambda}{}^{\tau\sigma}). \tag{19}$$

For anyone of the cases, we have the relation,

$$P_{\lambda} = \int_{\Omega} h\mathcal{T}^{0}{}_{\lambda}\ d^{3}x \tag{20}$$

where P_0 is the energy, while P_i stand for the four-momentum components and the integration hypersurface Ω is defined by $x^0 = t =$ constant. We remark that, for our purposes, it is not necessary to know the explicit form of the Einstein and Landau-Lifshitz gravitational energy-momentum pseudotensors. Instead, it is sufficient the relation between these complexes and their corresponding superpotentials, given by Eqs. (17) and (19).

3. The teleparallel homogeneous isotropic type solution

The line element of the homogeneous isotropic FRW universe is given by

$$ds^2 = dt^2 - \frac{a(t)^2}{(1+\frac{kr^2}{4})}(dr^2 + r^2d\theta^2 + r^2sin^2\theta d\phi^2), \tag{21}$$

where $a(t)$ is the time-dependent cosmological scale factor, and k is the curvature parameter $k = 0, \pm 1$. As remarked in Ref. [18], it is important to work with Cartesian

coordinates, as other coordinates may lead to non-physical values for the pseudotensor $t^{\tau}{}_{\lambda}$. Transforming, therefore, from polar to Cartesian coordinates, the FRW line element (21) becomes

$$ds^2 = dt^2 - \frac{a(t)^2}{(1+\frac{kr^2}{4})}(dx^2 + dy^2 + dz^2). \tag{22}$$

Using the relation (4), we obtain the tetrad components:

$$h^a{}_\mu = \mathrm{diag}\left(1,\ -\frac{a(t)}{1+\frac{kr^2}{4}},\ -\frac{a(t)}{1+\frac{kr^2}{4}},\ -\frac{a(t)}{1+\frac{kr^2}{4}}\right). \tag{23}$$

Its inverse is

$$h_a{}^\mu = \mathrm{diag}\left(1,\ -\frac{1+\frac{kr^2}{4}}{a(t)},\ -\frac{1+\frac{kr^2}{4}}{a(t)},\ -\frac{1+\frac{kr^2}{4}}{a(t)}\right). \tag{24}$$

One can easily verify that the relations (3) and (4) between $h^a{}_\mu$ and $h_a{}^\mu$ are satisfied.

From Eqs.(23) and (24), we can now construct the Weitzenböck connection, whose nonvanishing components are found

$$\begin{aligned}
\Gamma^x{}_{xt} &= \Gamma^y{}_{yt} = \Gamma^z{}_{zt} = \frac{\dot{a}(t)}{a(t)},\\
\Gamma^x{}_{xx} &= \Gamma^y{}_{yx} = \Gamma^z{}_{zx} = -\frac{kx}{2(1+\frac{kr^2}{4})},\\
\Gamma^x{}_{xy} &= \Gamma^y{}_{yy} = \Gamma^z{}_{zy} = -\frac{ky}{2(1+\frac{kr^2}{4})},\\
\Gamma^x{}_{xz} &= \Gamma^y{}_{yz} = \Gamma^z{}_{zz} = -\frac{kz}{2(1+\frac{kr^2}{4})}.
\end{aligned}$$

where a dot denotes a derivative with respect to the time t. The corresponding non-vanishing torsion components are:

$$\begin{aligned}
T^x{}_{tx} &= T^y{}_{ty} = T^z{}_{tz} = \frac{\dot{a}(t)}{a(t)},\\
T^x{}_{xy} &= T^z{}_{zy} = \frac{ky}{2(1+\frac{kr^2}{4})},\\
T^x{}_{xz} &= T^y{}_{yz} = \frac{kz}{2(1+\frac{kr^2}{4})},\\
T^y{}_{yx} &= T^z{}_{zx} = \frac{kx}{2(1+\frac{kr^2}{4})}
\end{aligned}$$

Now, the non-zero components of the tensor $S_\nu{}^{\sigma\tau}$ read

$$\begin{aligned}
S_t{}^{tx} &= S_y{}^{yx} = S_z{}^{zx} = -\frac{kx}{2a^2}\left(1+\frac{kr^2}{4}\right)\left(c_1+\frac{c_2}{2}+c_3\right),\\
S_t{}^{ty} &= S_x{}^{xy} = S_z{}^{zy} = -\frac{ky}{2a^2}\left(1+\frac{kr^2}{4}\right)\left(c_1+\frac{c_2}{2}+c_3\right),\\
S_t{}^{tz} &= S_x{}^{xz} = S_y{}^{yz} = -\frac{kz}{2a^2}\left(1+\frac{kr^2}{4}\right)\left(c_1+\frac{c_2}{2}+c_3\right),
\end{aligned}$$

$$S_x{}^{xt} = S_y{}^{yt} = S_z{}^{zt} = -\frac{\dot{a}(t)}{a(t)}\left(c_1 + \frac{c_2}{2} + c_3\right).$$

In more compact form, they are

$$U_t{}^{\sigma\tau} \equiv hS_t{}^{\sigma\tau} = -\frac{1}{2}\left(\delta_t^\sigma \delta_i^\tau - \delta_t^\tau \delta_i^\sigma\right) \frac{ka(t)x^i}{(1+\frac{kr^2}{4})} \left(c_1 + \frac{c_2}{2} + c_3\right). \tag{25}$$

Let us now calculate the total energy of the FRW universe at the instant $x^0 = t =$ constant, which is given by the integral over the space section. As in Ref [9], we carry out the integration in polar coordinates. For the Einstein energy-momentum complex, using Eqs. (17), (20) and (25), we have

$$E = k\frac{a(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{1}{2}\int_0^\infty \frac{kr^4dr}{(1+\frac{kr^2}{4})^3} - \frac{3}{2}\int_0^\infty \frac{r^2dr}{(1+\frac{kr^2}{4})^2}\right]. \tag{26}$$

For a closed universe ($k = +1$), the energy is

$$\begin{aligned} E &= \frac{a(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{1}{2}\int_0^\infty \frac{r^4dr}{(1+\frac{r^2}{4})^3} - \frac{3}{2}\int_0^\infty \frac{r^2dr}{(1+\frac{r^2}{4})^2}\right] \\ &= \frac{a(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{1}{2}6\pi - \frac{3}{2}2\pi\right] = 0, \end{aligned} \tag{27}$$

which is the same as Rosen's results [9]. For an open universe ($k = -1$), we obtain

$$\begin{aligned} E &= \frac{a(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{3}{2}\int_0^\infty \frac{r^2dr}{(1-\frac{r^2}{4})^2} + \frac{1}{2}\int_0^\infty \frac{r^4dr}{(1-\frac{r^2}{4})^3}\right] \\ &= \frac{a(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{3}{2}(\mp 2\pi i) + \frac{1}{2}(\pm 6\pi i)\right] = 0. \end{aligned} \tag{28}$$

Finally, for the spatially flat universe ($k = 0$), Eq. (26) gives again $E = 0$.

Now, for the Landau-Lifshitz complex, using (19), (20) and (25), the total energy is found to be

$$E = k\frac{a^4(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{5}{4}\int_0^\infty \frac{kr^4dr}{(1+\frac{kr^2}{4})^6} - \frac{3}{2}\int_0^\infty \frac{r^2dr}{(1+\frac{kr^2}{4})^5}\right]. \tag{29}$$

The energy of a closed universe ($k = +1$) is

$$\begin{aligned} E &= \frac{a^4(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{5}{4}\int_0^\infty \frac{r^4dr}{(1+\frac{r^2}{4})^6} - \frac{3}{2}\int_0^\infty \frac{r^2dr}{(1+\frac{r^2}{4})^5}\right] \\ &= \frac{a^4(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{5}{4}\frac{3\pi}{16} - \frac{3}{2}\frac{5\pi}{32}\right] = 0, \end{aligned} \tag{30}$$

which confirms the Johri *et al.* results [10]. For an open universe ($k = -1$),

$$\begin{aligned} E &= \frac{a^4(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{3}{2}\int_0^\infty \frac{r^2dr}{(1-\frac{r^2}{4})^5} + \frac{5}{4}\int_0^\infty \frac{r^4dr}{(1-\frac{r^2}{4})^6}\right] \\ &= \frac{a^4(t)}{G}\left(c_1 + \frac{c_2}{2} + c_3\right)\left[\frac{3}{2}\left(\mp\frac{5\pi i}{32}\right) + \frac{5}{4}\left(\pm\frac{3\pi i}{16}\right)\right] = 0. \end{aligned} \tag{31}$$

And finally, for the spatially flat universe ($k = 0$), Eq. (29) gives again $E = 0$.

4. Final Remarks

Working in the context of teleparallel gravity, we have calculated in the total energy of the FRW universe, which includes the energy of the matter and that of the gravitational field. In order to compute the gravitational part of the energy, we have considered the teleparallel version of both Einstein and Landau-Lifshitz energy-momentum complexes. Our basic result is that the total energy vanishes whatever be the pseudotensor used to describe the gravitational energy. It is also independent of both the curvature parameter and the three teleparallel dimensionless coupling constants. It is valid, therefore, not only in the telaparallel equivalent of general relativity, but also in any teleparallel model.

Finally, it is important to remark that, for an open FRW universe, the result that the total energy vanishes is quite *unexpected*. In fact, as the open FRW universe is an infinite spacetime filled with matter and gravitational field, our common sense would point to an infinite total energy. However, the result obtained is that the total energy for this universe is zero. We can thus conclude that the gravitational energy exactly cancels out the matter energy, and that this cancellation is independent of the curvature parameter.

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